Part 3

Introduction to Turbulence Model

The main field of application of Navier-Stokes methods in aerodynamics will be for complex turbulent flows that cannot be treated by inviscid or viscous-inviscid interaction schemes. Examples are highly separated flows, flows consisting of multiple length-scales, flows with three-dimensional separation and complex unsteady flows. In these cases, the application of algebraic turbulence models, like the Cebeci- Smith, the Baldwin-Lomax or the Johnson-King model, becomes very complicated and often ambiguous, mainly because of the difficulty to define an algebraic length-scale.

The most popular nonalgebraic turbulence models are two-equation eddy-viscosity models.

The k - ε model has been very successful in a vast variety of different flow situations, but it also has a numerous well known drawbacks. Here it is not aimed to only bring the formulation of two equation models even the Reynolds stress models but the main focus of this chapter is to discuss the ability of each and every single turbulence model and the reputation of each model in prediction of transition.

From the standpoint of aerodynamics, the most disturbing of this two equation models is the lack of sensitivity to adverse pressure-gradients. Another problem with the k- ε model is associated with the numerical stiffness of the equations when integrated through the viscous sublayer. All low Reynolds number k - ε models employ damping functions in different forms in the sublayer. These are generally highly nonlinear functions of dimensionless groups of the dependent variables which brings some numerical stiffness in the computation.

There are several alternative models that have been developed to tackle the shortcomings of the k - ε model. One of the most successful, with respect to both, accuracy and robustness, is the k - ω model of Wilcox.

The model does not employ damping functions and has straightforward Dirichlet boundary conditions. This results in important advantages in numerical stability.

Reportedly the performance of this model is significantly better under adverse pressure-gradient conditions than the k - ε model although it is the author's experience that it also prevents and delays the turbulence onset.

The most outstanding shortcoming of *k – ω model* is its dependence on freestream value of ω_{f}, which is specified outside the shear-layer.

New baseline k- ω model is developed to address the problem of Wilcox k- ω model for its dependency on free stream value of ω_f. In fact this model is the blend of original k- ω model near the wall and k-ε model outside the boundary layer. The performance of baseline model is very similar of that Wilcox model but without that undesirable freestream dependency. Although the original - and the new BSL k - ω, model perform better in adverse pressure gradient flows than the k - ε model, they still underpredict the amount of separation for severe adverse pressure gradient flows.

The other turbulence model that has been implemented in the code and some computation is carried out with the aid of that turbulence model is Wallin and Johansson algebraic stress model. This is in fact an explicit algebraic stress model (EARSM) which is classified under two equation models in TAU code.

Non-linear EVM have been suggested to overcome the shortcomings of linear EVM while retaining their benefits with respect to stability and convergence. Explicit algebraic stress models are a class of non-linear EVM.

In this model the Reynolds stresses are modeled by effective turbulent viscosity and an extra anisotropy $a_{ij}^{(ex)}$

$$-r\,\overline{u_i u_j} = 2m_t S_{ij}^* - \frac{2}{3} r\,k\,d_{ij} - r\,k a_{ij}^{(ex)}$$

The extra anisotropy factor is of course taken into account because of uncertainty of simple eddy viscosity model (EVM) which simply represents the turbulence by its kinetic energy and seemingly can not properly represent turbulence anisotropy.

2. Second Moment Closure Models

The aforementioned fact was that two equation models and more specifically the eddy viscosity models are unable to carry the burden of anisotropy of the turbulence due to their simplicity in representing the turbulence by kinetic energy and making it proportional by a constant, defined as eddy viscosity models. These models are inaccurate for flows with sudden changes in their mean strain rate. Moreover these two equation models are unable to predict a good estimate for flows with curved surface, flows in duct with secondary motions, flow in rotating flows and flow with boundary layer separation.

As it was stated in previous section, one remedy is to use non-linear constitutive equation for eddy viscosity model. More accurately, the approach to achieve a more appropriate description of Reynolds stress tensor is to assume Boussinesq approximations as simply the leading term in series expansion of functionals.

The other approach is to completely discard the EVM formulation and solve the equation for Reynolds stress terms. This is called Reynolds stress model which is a higher level and more elaborate turbulence model and usually is called second moment closure model because of the necessity of existence of second equation of length scale to close the exact stress transport equation. The derivation and modeling of each term of RSM is out of scope of this section.

In this thesis work two Reynolds stress model has been used; the original Reynolds stress model of Wilcox and *LRR-SSG* Reynolds stress model.

The term *RSM of Wilcox* which is defined in the code in this name, is originally Wilcox Stress-ω model that postulate a stress-transport model based on ω equation as length scale. This is actually the main difference between the LRR-SSG and stress- ω models that in the later one the ω equation is used to compute dissipation and in the former, Baseline of Menter has been recruited as a length scale equation.

The high-Reynolds number compressible version of the stress- ω model is as follow

Reynolds Stress Tensor

$$r\frac{\partial t_{ij}}{\partial t} + rU_k\frac{\partial t_{ij}}{\partial x_k} = -rP_{ij} + \frac{2}{3}b^{*}rwk\,d_{ij} - r\Pi_{ij} + \frac{\partial}{\partial x_k}[(m + s^{*}m_t)\frac{\partial t_{ij}}{\partial x_k}] \tag{7}$$

Specific Dissipation Rate

$$r\frac{\partial w}{\partial t} + rU_j\frac{\partial w}{\partial x_j} = a\frac{rw}{k}t_{ij}\frac{\partial U_i}{\partial xj} - brw^2 + \frac{\partial}{\partial x_k}[(m + sm_t)\frac{\partial w}{\partial x_k}] \tag{8}$$

Pressure-Strain Correlation

$$\Pi_{ij} = b^{*} C_1 w (t_{ij} + \frac{2}{3} k d_{ij}) - a (P_{ij} - \frac{2}{3} P d_{ij}) - b (D_{ij} - \frac{2}{3} P d_{ij}) - g k (S_{ij} - \frac{1}{3} S_{kk} d_{ij}) \qquad (9)$$

Auxiliary Relations

$$m_t = r k / w$$

$$P_{ij} = t_{im} \frac{\partial U_j}{\partial x_m} + t_{jm} \frac{\partial U_i}{\partial x_m}$$

$$D_{ij} = t_{im} \frac{\partial U_m}{\partial x_j} + t_{jm} \frac{\partial U_m}{\partial x_i}$$

$$P = \frac{1}{2} P_{kk}$$